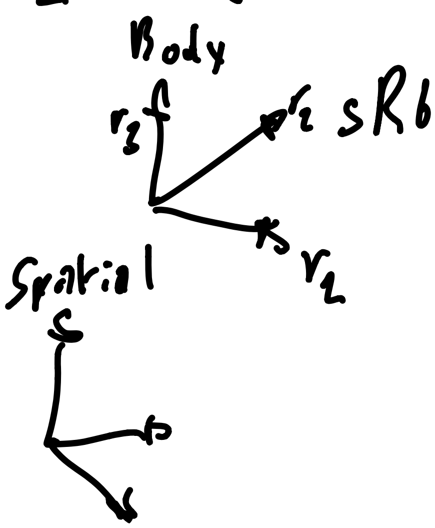
Plan:

· Notes
· Lynch & Park 17 pdf online

1. SO(3) . Rot in 3D
2. SE(3) . Transforms in 3D –
3. Adjoints . $V_a = Ad_{{}_aT_s} V_s$
4. M + A . Home position + Delta $\rightsquigarrow$ Dynamics
5. POE . Product of Exponentials $\rightarrow$ Man. Jac.

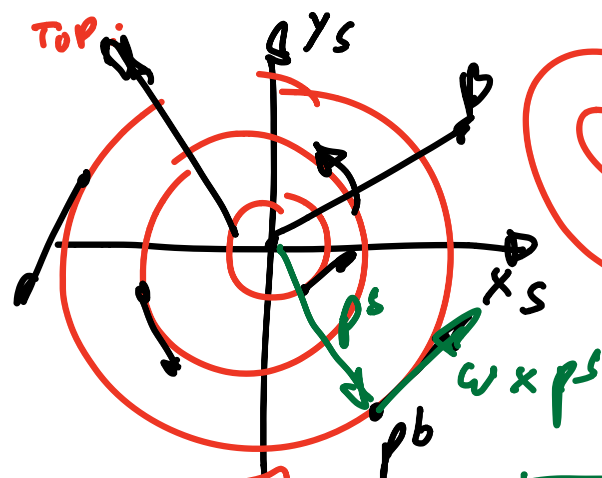
$$\boxed{SO(3) \qquad p_s = ? \qquad \dot{p}_s = ? \qquad p_s(t) = ?}$$

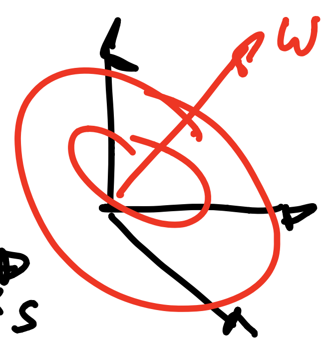
$R = [r_1 \; r_2 \; r_3]$



$$\boxed{p^s = {}_sR_b \, p^b} \qquad p^s = R^s_{\cancel{b}} p^b$$

$$p^s = {}_sR_b(t) \, p^b$$





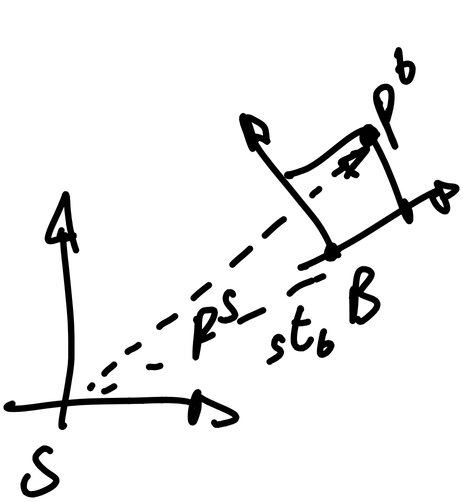
$$\boxed{\dot{p}^s = {}_s\dot{R}_b \, p^b = \omega \times p^s} = \underbrace{{}_s\dot{R}_b \; {}_sR_b^{-1}}_{} p^s$$

$$\begin{bmatrix} 0 & \cdot & \cdot \\ \cdot & 0 & \cdot \\ \cdot & \cdot & 0 \end{bmatrix} = \hat{\omega} = [\omega]$$

$$= \omega \times p^s$$

$$\boxed{SE(3) \qquad p_s = ? \qquad \dot{p}_s = ? \qquad p_s(t) = ?}$$

$$\tilde{p}^s = {}_sT_b \, \tilde{p}^b = \begin{bmatrix} {}_sR_b & {}_st_b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p^b \\ 1 \end{bmatrix}$$



$$p^s = {}_sT_b(t) \, p^b$$

$$\dot{p}^s = \underbrace{{}_s\dot{T}_b \; {}_sT_b^{-1}}\, p^s$$

$$\begin{bmatrix} \dot{p}^s \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix}}_{\hat{V}^s} \begin{bmatrix} p^s \\ 1 \end{bmatrix}$$

$$V^s = \begin{bmatrix} \omega \\ v \end{bmatrix} \in \mathbb{R}^6$$

$$so(3) \longleftrightarrow \mathbb{R}^3$$
$$\hat{\omega} \longleftarrow \omega$$

$$se(3) \longleftrightarrow \mathbb{R}^6$$
$$\hat{V} \longleftarrow V$$

$$\dot{p}^s = \hat{\omega} p^s + v$$
$$\underline{\dot{p}^s} = \underline{\omega \times p^s} + \underline{v}$$

$$p_s(t) = ?$$

$$\underbrace{\dot{\tilde{p}}^s}_{4\times 1} = \underbrace{\hat{V}^s}_{4\times 4} \cdot \underbrace{\tilde{p}^s}_{4\times 1}$$

$$\dot{x} = ax \qquad x(t) = e^{at} x_0$$

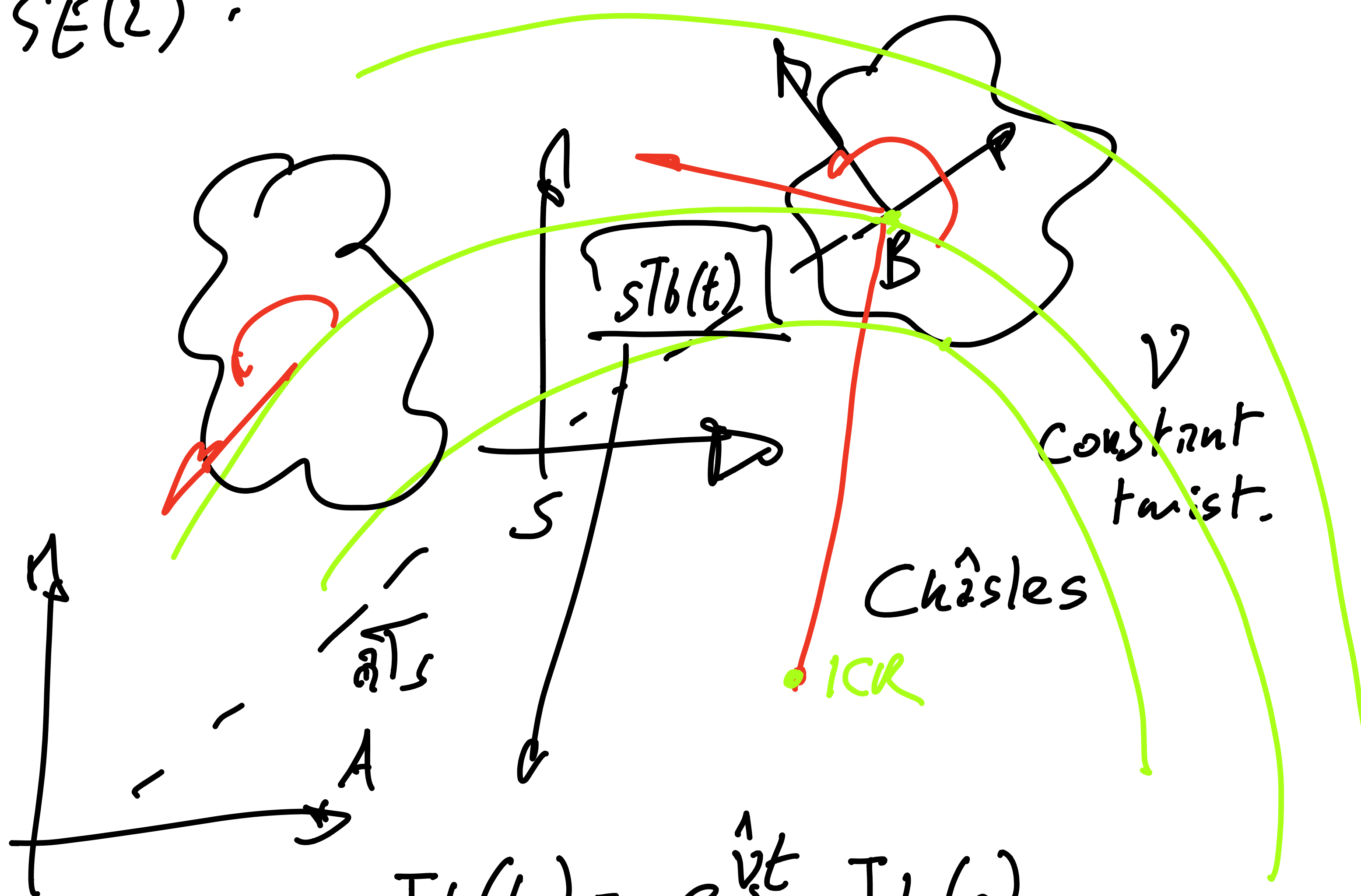
$$p^s(t) = \underbrace{e^{\hat{V}^s t}}_{\text{exp. map}} \, p^s(0)$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\boxed{e^A = I + \frac{A}{1!} + \frac{A^2}{2!} + \frac{A^3}{3!} \dots}$$

$\boxed{\text{Adjoints} \quad \mathcal{V}_s \to \mathcal{V}_a \ ?}$

$SE(2)$:



$$sTb(t) = e^{\hat{\mathcal{V}}_s t}\, sTb(0)$$

$$\boxed{p^a = aTs\, p^s}$$

$$p^s(t) = e^{\hat{\mathcal{V}}_s t}\, p^s(0)$$

$$p^a(t) = e^{\hat{\mathcal{V}}_a t}\, p^a(0)$$

$$aTs\, p^s(t) = e^{\hat{\mathcal{V}}_a t}\, aTs\, p^s(0)$$

$$p^s(t) = aTs^{-1}\, e^{\hat{\mathcal{V}}_a t}\, aTs\, p^s(0)$$

$$= e^{{}_aT_s^{-1}\hat{V}_a t\, {}_aT_s} p^s(0) \qquad {}_aT_s = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$$

$$\hat{V}_s = {}_aT_s^{-1}\,\hat{V}_a\,{}_aT_s \qquad V_a \in \mathbb{R}^6 \text{ as a f. of } V_s$$

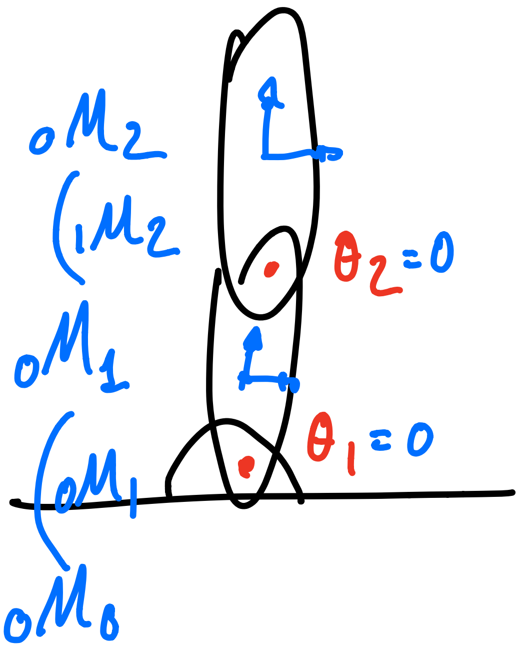
$$\boxed{\hat{V}_a = {}_aT_s\,\hat{V}_s\,{}_aT_s^{-1}} \longleftarrow V_a = \begin{bmatrix} R & 0 \\ \hat{t}R & R \end{bmatrix} V_s = Ad_{{}_aT_s} V_s$$

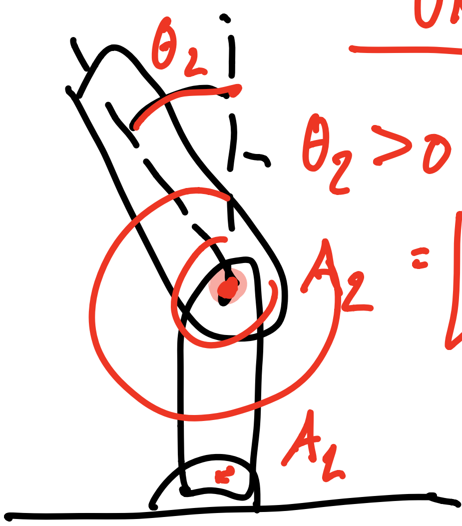
se(3) se(3)

$\boxed{\text{Home } M + \text{axes } A \quad , \quad FG}$

Chapt. 8 L & P ⟶ Chap 5 POE

UNIT TWIST





$= \begin{bmatrix} \bar{\omega} \\ p\times\bar{\omega} \end{bmatrix}$

$_0T_2$

$_0T_1$

$_0T_0 = I$

$${}_1M_2 = {}_0M_1^{-1}\,{}_0M_2 \quad \text{"between"}$$

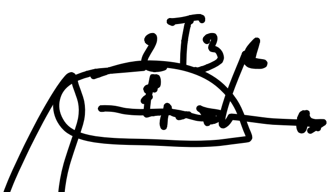
$${}_1T_2 = {}_0T_1^{-1}\,{}_0T_2$$

$$\boxed{{}_1T_2 = {}_1M_2\, e^{\hat{A}_2\theta_2}}$$

$$\boxed{{}_0T_2 = {}_0T_1\,{}_1M_2\,e^{\hat{A}_2\theta_2}}$$

4 x 4 $I \rightarrow$

AT REST Δ



POE

Chapter 8

config

$$
\boxed{
\begin{aligned}
{}_0T_3 &= {}_0T_1\;{}_1T_2\;{}_2T_3 \\
&= {}_0M_1\, e^{\hat{A}_1\theta_1}\; {}_1M_2\, e^{\hat{A}_2\theta_2}\; {}_2M_3
\end{aligned}
}
\qquad \xrightarrow[\text{wrpt}]{\exp} M
$$

$$= \underbrace{{}_0M_1\, e^{\hat{A}_1\theta_1}\, {}_0M_1^{-1}}\; \underbrace{{}_0M_2\, e^{\hat{A}_2\theta_2}\, {}_0M_2^{-1}}\; {}_0M_3$$

$$= e^{{}_0M_1 \hat{A}_1\, {}_0M_1^{-1}\theta_1}\; e^{Ad_{{}_0M_2} A_2}\; {}_0M_3$$

$$\boxed{{}_0T_3(q) = e^{\hat{S}_1\theta_1} * e^{\hat{S}_2\theta_2} * {}_0M_3}$$

Ch 5

GTSAM Pose3::Expmap

$\mathbb{R}^6 \to$ Pose3

001

001

001

Unit twist

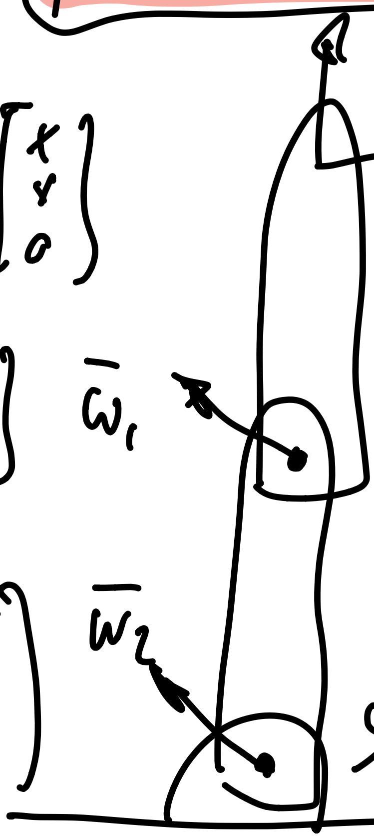
100

100

$p_1 = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\bar{\omega}_1$

$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\bar{w}_2$



$\to {}_0M_3$

$$S_2 = \begin{bmatrix} \bar{\omega}_2 \\ \bar{\omega}_2 \times p_2 \end{bmatrix} \in \mathbb{R}^6$$

$$S_1 = \begin{bmatrix} \bar{\omega}_1 \\ \bar{\omega}_1 \times p_1 \end{bmatrix} \in \mathbb{R}^6$$

$\angle \alpha\beta$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} +y \\ -x \\ 0 \end{bmatrix} \quad p^{\perp}$$